Three Dimensional Geometry

- Relation between the direction cosines of a line $\boxed{l^2 + m^2 + n^2 = 1}$ direction cosines

- Direction cosines of a line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\boxed{\frac{x_2 - x_1}{PQ} = \frac{y_2 - y_1}{PQ} = \frac{z_2 - z_1}{PQ}}$$

where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Vector Equation of a line that passes through the given point whose position vectors $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\boxed{\vec{r} = \vec{a} + \lambda \vec{b}}$ (vector form)

Cartesian Equation,

$$\boxed{\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}} \quad \text{OR} \quad \boxed{\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}}$$

direction ratios

- Vector Equation of a line that passes through two points $\boxed{\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})}$, $\lambda \in R$

position vectors

Cartesian Equation points :- $(x_1, y_1, z_1)\ (x_2, y_2, z_2)$ $\boxed{\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}}$

- If $a_1; b_1; c_1$ and $a_2; b_2; c_2$ are the direction ratios of two lines and θ is the acute angle between two lines then;

$$\boxed{\cos\theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|}$$

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are

(i) perpendicular $\theta = 90^\circ$ $\boxed{a_1 a_2 + b_1 b_2 + c_1 c_2 = 0}$

(ii) parallel $\theta = 0^\circ$ $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

If θ is the acute angle between the line $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$; then θ is given by :

$$\boxed{\cos\theta = \left| \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|} \right|} \quad \text{OR} \quad \boxed{\theta = \cos^{-1} \frac{|\vec{b_1} \cdot \vec{b_2}|}{|\vec{b_1}||\vec{b_2}|}}$$

(vector form)

- The shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$ is

$$\boxed{\left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|}$$

Cartesian form, lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$ is

$$\boxed{\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}}$$

- Distance between Parallel lines $\vec{r} = \vec{a_1} + \mu \vec{b}$ and $\vec{r} = \vec{a_2} + \mu \vec{b}$ is $\boxed{\left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|}$

- Equation of a plane in a normal form $\boxed{\vec{r} \cdot \hat{n} = d}$ (vector form)

position vector; unit normal vector; distance from origin

Cartesian form, $\boxed{lx + my + nz = d}$

- The equation of a plane through a point whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $\boxed{(\vec{r}-\vec{a})\cdot\vec{N}=0}$ (vector form)

  Cartesian form, $\boxed{A(x-x_1)+B(y-y_1)+C(z-z_1)=0}$

- Equation of a plane passing through three non collinear points

  $\boxed{(\vec{r}-\vec{a})[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0}$ (vector form)

  Cartesian form, $\boxed{\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0}$

- Plane passing through the intersection of two given planes $\boxed{\vec{r}\cdot(\vec{n_1}+\lambda\vec{n_2})=d_1+\lambda d_2}$ (vector form)

  Cartesian form, $\boxed{(A_1x+B_1y+C_1z-d_1)+\lambda(A_2x+B_2y+C_2z-d_2)=0}$

- Angle between two planes $\boxed{\cos\theta=\left|\dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|}$

  Cartesian form $\boxed{\cos\theta=\left|\dfrac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|}$

- Angle between a line and a plane $\boxed{\cos\theta=\left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|}$

  the angle ϕ between the line and the plane is given by $90^\circ-\theta$, i.e. $\sin(90^\circ-\theta)=\cos\theta$

  $\boxed{\sin\phi=\left|\dfrac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|}$ OR $\boxed{\phi=\sin^{-1}\left|\dfrac{\bar{b}\cdot\bar{n}}{|\bar{b}|\cdot|\bar{n}|}\right|}$